Hassenstein-Reichardt Correlator is a transformer with a heterogeneous value function

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Here, we reformulate the mathematical expression of the famous Hassenstein-Reichardt Correlator (HRC) for the motion detection [1, 2], and show that it looks like a transformer with a heterogeneous value function. For simplicity, we only consider the motions in one dimensional space, where the visual signal is a function of the space and time $s(x,t)$. In a standard HRC, there are two receptors at two nearby locations in space, and we represent the visual signals received by the two receptors as $s_1(t)$ (the left one) and $s_2(t)$ (the right one), respectively. The HRC has the following form:

$$\begin{aligned}\hat{v} &= \int_0^T dt f(t)s_1(t)\int_0^T dt' g(t')s_2(t') - \int_0^T dt\ g(t)s_1(t)\int_0^T dt'\ f(t')s_2(t') \\ &= \int_0^T\int_0^T dtdt'\left[f(t)g(t') - f(t')g(t)\right]s_1(t)s_2(t'),\end{aligned}$$

where $f(t)$ and $g(t)$ are two temporal filters, and T is the length of each input signal. Usually, if we define that the rightward motion has positive velocity and that the leftward motion has negative velocity, $f(t)$ could be a low pass filter and $g(t)$ could be a high pass filter. To compare the above equation with the standard transformer architecture, we do the following discretization:

$$\hat{v} = \sum_{i=1}^n\sum_{j=1}^n \Delta t\Delta t'\left[f(i\Delta t)g(j\Delta t') - f(j\Delta t')g(i\Delta t)\right]s_1(i\Delta t)s_2(j\Delta t'),$$

where $\Delta t = \Delta t' = T/n$. For simplicity, we set $\Delta t = \Delta t' = 1$. We define $V_{ij} = f(i)g(j) - f(j)g(i)$, $s_1(i) = q_i$ and $s_2(j) = k_j$. With these arrangements, we have

$$\hat{v} = \sum_{i=1}^n\sum_{j=1}^n V_{ij}q_ik_j.$$

Now, we can define $\alpha_{ij} = q_i k_j$, and have

$$\hat{v} = \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} \alpha_{ij}.$$

Thus, HRC has a mathematical structure that is similar to a transformer with a heterogeneous value function V.

References

[1] Bernhard Hassenstein and Werner Reichardt. Systemtheoretische analyse der zeit-, reihenfolgen-und vorzeichenauswertung bei der bewegungsperzeption des rüsselkäfers chlorophanus. *Zeitschrift für Naturforschung B*, 11(9-10):513–524, 1956.

[2] Edward H Adelson and James R Bergen. Spatiotemporal energy models for the perception of motion. *Journal of the optical society of america A*, 2(2):284–299, 1985.